TRAVELING POLES ELIMINATION SCHEME AND CALCULATIONS OF EXTERNAL QUALITY FACTORS OF HOMS IN SC CAVITIES*

T. Galek†, T. Flisgen and U. van Rienen, Rostock University, Rostock, Germany
B. Riemann, Technical University Dortmund, Dortmund, Germany
A. Neumann, Helmholtz-Zentrum Berlin, Berlin, Germany

Abstract

The main scope of this work is the automation of the extraction procedure of the external quality factors Q_{ext} of Higher Order Modes (HOMs) in Superconducting (SC) radio frequency (RF) cavities. The HOMs are generated by charged particle beams traveling through a SC cavity at the speed of light ($\beta \approx 1$). The HOMs decay very slowly, depending on localization inside the structure and cell-to-cell coupling, and may influence succeeding charged particle bunches. Thus it is important, at the SC cavity design optimization stage, to calculate the Q_{ext} of HOMs. Traveling Poles Elimination (TPE) scheme was used to automatically extract Q_{ext} from the transmission spectra and careful eigenmode analysis of the SC cavity was performed to confirm TPE results. The eigenmode analysis also delivers important information about band structure, cell-to-cell coupling and allows rapid identification of modes that could interact with the charged particle bunches.

INTRODUCTION

The SC RF cavity presented in this article is a 1.3 GHz 7-cell Cornell design modified TESLA cavity with JLab HOM waveguide couplers as shown in figure 1. The discussed SC RF cavity will be used in the Berlin Energy Recovery Linac Project (B*ERL*inPro), which is currently under development for a CW LINAC technology and expertise required to drive next-generation Energy Recovery Linacs (ERLs) [1]. The main priority on the current stage of the cavity design requires strongly damped HOMs in order to obtain high performance of the linac.

Using a modern simulation software one can efficiently calculate all the necessary quantities during the optimization steps. Simulations used to obtain results presented in this article can be divided into two main categories: eigenmode simulations and frequency domain simulations. The eigenmode simulations give us important information about all the modes that can exist in the model structure in the given frequency range. Important quantities can be calculated as a post processing step, e.g. R/Q which is a measure of a mode interaction with the charged particle beam, E_{peak}/E_{acc} and H_{peak}/E_{acc} which are relevant to suppression of field emission and thermal break down. The frequency domain simulations are used to obtain S-parameter spectra from which Q_{ext} factors of HOMs can be extracted. For this purpose we present an automated procedure that is using vector fitting with rational functions to express the S-parameter transmission spectra with a set of poles. The Traveling Poles Elimination (TPE) scheme is a simple iterative procedure which main purpose is to detect static poles and calculate external quality factors. All the simulations were performed using CST Microwave Studio 2012 (CST MWS) [2].

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† tomasz.galek@uni-rostock.de

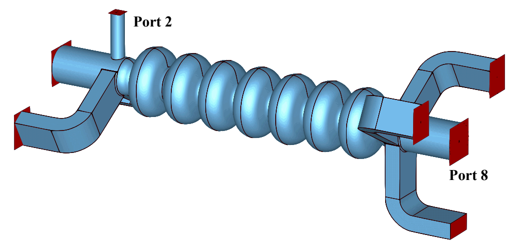

Figure 1: 7-cell TESLA cavity with coaxial input and HOM waveguide couplers

POLE FITTING

Rational Fitting of S-Parameter Spectra

For the extraction of the external quality factors Q_{ext} from S-parameter spectrum the fast implementation of the Vector Fitting (VF) algorithm was used [3]. The vector fitting is an iterative procedure of pole relocation by solving a linear least squares problem until the convergence criterion is met. The VF employs a method to ensure stable poles by flipping unstable poles into the left half complex plane. To achieve a faster convergence the algorithm uses, during the pole identification step, a relaxed non-triviality constraint and utilizes the matrix structure [4, 5, 6].

The S-parameter spectra are assumed to follow the complex rational function approximation

$$S(f) = \sum_{k=1}^{N} \frac{a_k}{2\pi i f - p_k} + R_k, \tag{1}$$

where $i^2 = -1$ is the imaginary unit, a_k the residues, p_k complex conjugate pairs of poles and R_k a frequency-independent residual summarizing all other contributions. The complex pole $p_k = \alpha_k + i\omega_k$ contains a resonance frequency $\omega_k = 2\pi f_k$ and an attenuation constant α_k. The quality factor Q_k for a given pole can be obtained using

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$$Q_k = -\frac{\Im(p_k)}{2\Re(p_k)} = -\frac{\omega_k}{2\alpha_k}. \tag{2}$$

A purely imaginary pole ($\alpha = 0$) would correspond to an infinite quality factor, thus any sharp peak appearing in the S-parameter spectrum will have very high Q_{ext}. This applies to passbands with very narrow bandwidth as well.

Traveling Poles Elimination Scheme

The Traveling Poles Elimination (TPE) scheme is a simple iterative procedure using the VF algorithm, with a goal to detect static poles among a set of unstable poles. The TPE procedure can be summarized in few steps:

- The first TPE iteration uses a set of starting poles. The number and location of the starting poles depends on number of peaks in the S-parameter spectrum. A simple peak finding procedure automatically finds peaks and assigns starting complex conjugate pair of poles to each peak.
- In subsequent TPE iterations the set of fitted poles from the previous TPE iteration is expanded by a number of additional pairs of poles added randomly. The number of freshly added poles should not exceed 10% of the total number of poles from previous TPE iteration.
- When all the TPE iterations are done, the poles are sorted and close neighbors are detected. The set of poles from the last TPE iteration is taken as a target set. For each pole in the target set, the sorting procedure is searching for close neighbors in pole sets from the previous TPE iterations. The closest neighbors are found within frequency TOL_f and quality factor TOL_Q tolerances defined by the user.

Validation of the TPE Procedure

The TPE procedure was validated on a simpler model of the 7-cell cavity. All the geometrical parameters were kept the same only the test structure consists of two end-cells, and the same port setup, as used for the S-parameter simulations. The frequency domain simulations were performed using fast resonant frequency solver in CST MWS, using hexahedral mesh with 15 steps per wavelength, resulting in total of 1.5 million mesh cells, and frequency range 1.2 - 3.3 GHz. The spectrum used for validation is a transmission spectrum S8(3)2(1), notation used is similar to that of CST MWS, from coaxial port 2 using TEM mode 1 to the beam pipe port 8 using TM_{01} monopole mode 3. Additionally to cross check the results, eigenmode simulations were performed for the 2-cell test cavity using JDM eigenmode solver in CST MWS, which allows to calculate quality factors for all the modes.

Figure 2 shows the S8(3)2(1) transmission spectrum, Q_{ext} factors extracted using TPE procedure and Q_{ext} factors calculated using eigenmode solver. There are many eigenmode Q_{ext} factors not matched with static poles, these modes are mostly waveguide modes, starting to appear around 1.578 GHz (cutoff frequency of the first waveguide mode) and have low quality factors $Q_{ext} < 10^3$. The waveguide modes are irrelevant in S-parameter spectrum because these modes arise only in eigenmode simulations due to boundary conditions enforced by the eigenmode solver. The other poles found by the TPE procedure are in good agreement with the Q_{ext} factors from eigenmode calculations.

EIGENMODE ANALYSIS

The eigenmode analysis was performed in a similar way to the one presented by R. Wanzenberg for the 9-cell TESLA cavity [7]. Two separate eigenmode simulations for just a single cell, with periodic boundary conditions (PBC) at x-min and x-max limits, were computed. While using the PBC one can control the phase shift from one cell to the other, in this case it is 0° and 180° at the PBC. The eigenmode simulations were performed using a tetrahedral mesh with curved elements (2nd order). The frequency range was set to 1.2 - 3.3 GHz, and 26 modes per simulation were calculated.

What one can learn from such an approach is the resonance frequency of the fundamental 0-mode f_0 (0° phase shift at PBC) and the π-mode f_π (180° phase shift at PBC). The same rules apply to higher order modes. The passband width of a given mode is given by f_0 and f_π, and all the resonance frequencies of the modes within one band follow a cosine-like dispersion curve, and the number of modes in the band depends on the number of the cells in the cavity. To calculate the cell-to-cell coupling factor k_{cc} one needs only f_0 and f_π [7, 8]

$$k_{cc} = 2 \cdot \frac{f_\pi - f_0}{f_\pi + f_0}. \tag{3}$$

The k_{cc} factor can be either positive, i.e., $f_0 < f_\pi$, or negative, i.e., $f_0 > f_\pi$. In addition the k_{cc} factor specifies the passband width, smaller k_{cc} gives narrower passbands. For small ($|k_{cc}| \leq 0.01$) values there is a danger that if the given mode is excited by the beam, e.g., somewhere in the middle of the cavity, it will propagate out and decay very slowly. Thus the k_{cc} factor gives us a preliminary knowledge of which modes can be dangerous or trapped. In table 1 the results of the eigenmode analysis are gathered, including the f_0 and f_π of all the modes, mode type, cell-to-cell coupling k_{cc} factor, R/Q and Q_{ext} extracted from the 7-cell cavity S-parameter transmission spectra using TPE procedure.

The eigenmode analysis results are in good agreement with the 9-cell TESLA cavity [7] results. There are of course some differences in the band frequencies, due to the fact that it is a bit different geometry. There is one inconsistency though, an additional TE monopole mode (TE M1 in table 1, mode number 9 for 0° phase shift and mode number 13 for 180° phase shift) that is not in the report by R. Wanzenberg [7]. Additionally current eigenmode results contain three sextupole bands not included in [7]. The

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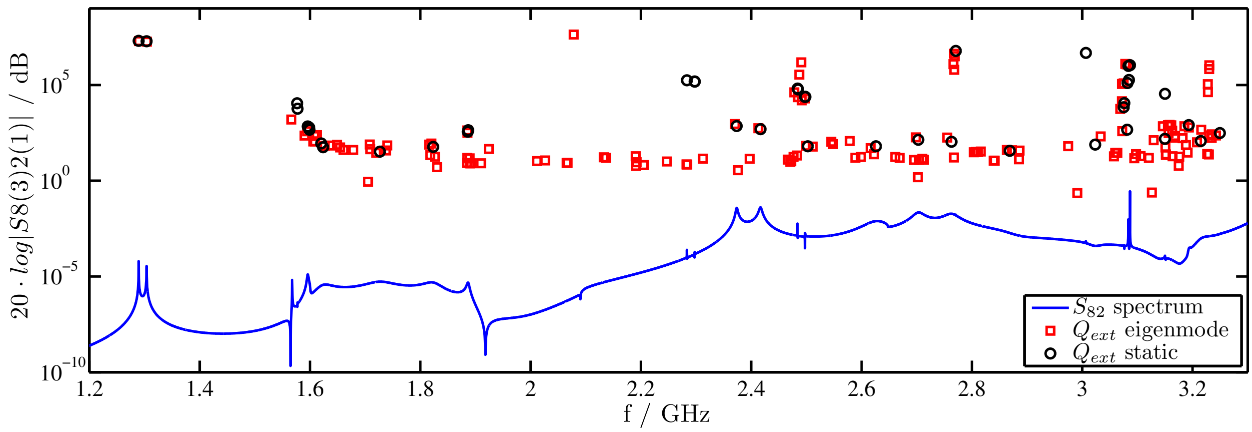

Figure 2: The S-parameter spectrum for the 2-cell test structure (blue solid line). The external quality factors were obtained during eigenmode simulations of the test structure (red squares) and using TPE procedure (black circles).

Table 1: Overview of the Eigenmode Analysis Results

		Phase advance 0°					Phase advance 180°				
Mode	**f/GHz**	**R/Q**	**Q_{ext}**	**R/Q Q_{ext}**	**Mode**	**f/GHz**	**R/Q**	**Q_{ext}**	**R/Q Q_{ext}**	**Type**	**k_{cc}**
1	1.273	1.44E+02	1.97E+07	2.83E+09	1	1.3	111.411	1.75E+07	1.95E+09	TM M1	0.021
2, 3	1.593	3.80E-09	1.29E+04	4.91E-05	4, 5	1.814	4.85E-08	5.82E+01	2.82E-06	TE D1	0.13
4, 5	1.888	2.56E-08	4.34E+02	1.11E-05	2, 3	1.782	7.97E-08	3.60E+01	2.87E-06	TM D1	-0.058
6, 7	2.278	4.17E-08	1.75E+05	7.31E-03	6, 7	2.304	1.91E-07	1.50E+05	2.87E-02	TE Q1	0.011
8	2.438	26.1306	4.92E+02	1.29E+04	8	2.354	23.7545	7.14E+02	1.70E+04	TM M2	-0.035
9	2.484	3.23E-08	1.81E+07	5.84E-01	13	2.49	1.04E-08	1.03E+07	1.08E-01	TE M1	0.0024
10, 11	2.492	3.09E-07	4.79E+04	1.48E-02	11, 12	2.467	8.53E-07	6.50E+04	5.54E-02	TM Q1	-0.01
12, 13	2.552	1.21E-07	7.80E+01	9.43E-06	9, 10	2.444	7.70E-08	7.20E+01	5.54E-06	TE D2	-0.043
14	2.675	0.416747	1.38E+02	5.76E+01	18	2.789	83.8406	1.00E+02	8.38E+03	TM M3	0.042
15, 16	2.765	1.21E-08	3.31E+07	3.99E-01	14, 15	2.768	2.81E-07	1.52E+07	4.26E+00	TE S1	0.0011
17, 18	3.012	1.38E-07	7.46E+01	1.03E-05	16, 17	2.785	8.37E-06	1.08E+02	9.02E-04	TE D3	-0.078
19, 20	3.075	3.65E-08	1.13E+04	4.11E-04	21, 22	3.071	2.02E-08	7.50E+03	1.52E-04	TM S1	-0.0013
21, 22	3.11	5.14E-07	9.80E+01	5.03E-05	19, 20	3.065	1.01E-06	2.86E+01	2.88E-05	TE D4	-0.015
23, 24	3.165	5.99E-07	2.84E+02	1.70E-04	25, 26	3.278	1.71E-08	2.84E+02	4.84E-06	TE Q2	0.035
25, 26	3.227	9.75E-11	2.83E+02	2.76E-08	23, 24	3.228	6.10E-09	2.83E+02	1.73E-06	TE S2	0.0003

R/Q values have been calculated on the beam axis for all the modes.

CONCLUSIONS

In this work we have presented a very simple numerical method to automatically extract external quality factors and resonance frequencies from the transmission S-parameter spectra. The traveling pole elimination scheme was validated on a simplified model of the cavity with only two cells. Test structure geometric dimensions were kept the same as for the full 7-cell cavity. Afterward the VF and TPE procedures were used to study real SC RF 7-cell TESLA cavity. The Q_{ext} factors were extracted for all the modes in 1.2 - 3.3 GHz frequency range. These results in combination with results obtained from eigenomde simulations give detailed insight into HOMs behavior in 1.2 - 3.3 GHz frequency range. The optimization of the 7-cell cavity for the B*ERL*inPro main linac requires investigations of HOMs in higher frequency ranges, thus the same methodology will be applied in the future.

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